WeBWorK 標準問題集: 線形代数学 B

WeBWorK Standard Problems: Linear Algebra B

June 20, 2023

Let $A = \begin{bmatrix} 5 & -2 \\ -4 & -3 \end{bmatrix}$.

(a) Find the determinant of A.

$\det(A) = \square$

(b) Find the matrix of cofactors of A.

$C = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

(c) Find the adjugate of A.

$\mathrm{adj}(A) = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

(d) Find the inverse of A.

$A^{-1} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

Let $A = \begin{bmatrix} 5 & -2 \\ -4 & -3 \end{bmatrix}$.

(a) Find the determinant of A.

$\det(A) = \boxed{-23}$

(b) Find the matrix of cofactors of A.

$C = \begin{bmatrix} \boxed{-3} & \boxed{4} \\ \boxed{2} & \boxed{5} \end{bmatrix}$

(c) Find the adjugate of A.

$\operatorname{adj}(A) = \begin{bmatrix} \boxed{-3} & \boxed{2} \\ \boxed{4} & \boxed{5} \end{bmatrix}$

(d) Find the inverse of A.

$A^{-1} = \begin{bmatrix} \boxed{3/23} & \boxed{-2/23} \\ \boxed{-4/23} & \boxed{-5/23} \end{bmatrix}$

Let $A = \begin{bmatrix} -2 & -2 & -3 \\ -2 & -3 & 1 \\ -3 & 0 & 3 \end{bmatrix}$.

(a) Find the determinant of A.

$\det(A) = \square$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

(c) Find the adjugate of A.

$$\mathrm{adj}(A) = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

(d) Find the inverse of A.

$$A^{-1} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

Let $A = \begin{bmatrix} -2 & -2 & -3 \\ -2 & -3 & 1 \\ -3 & 0 & 3 \end{bmatrix}$.

(a) Find the determinant of A.

$\det(A) = \boxed{39}$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} \boxed{-9} & \boxed{3} & \boxed{-9} \\ \boxed{6} & \boxed{-15} & \boxed{6} \\ \boxed{-11} & \boxed{8} & \boxed{2} \end{bmatrix}$$

(c) Find the adjugate of A.

$$\operatorname{adj}(A) = \begin{bmatrix} \boxed{-9} & \boxed{6} & \boxed{-11} \\ \boxed{3} & \boxed{-15} & \boxed{8} \\ \boxed{-9} & \boxed{6} & \boxed{2} \end{bmatrix}$$

(d) Find the inverse of A.

$$A^{-1} = \begin{bmatrix} \boxed{-3/13} & \boxed{-2/13} & \boxed{-11/39} \\ \boxed{1/13} & \boxed{-5/13} & \boxed{8/38} \\ \boxed{-3/13} & \boxed{2/13} & \boxed{2/39} \end{bmatrix}$$

Let $A = \begin{bmatrix} 3e^{2t} & 2e^{4t} \\ 4e^{2t} & -3e^{4t} \end{bmatrix}$.

(a) Find the determinant of A.

$\det(A) = \square$

(b) Find the matrix of cofactors of A.

$C = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

(c) Find the adjugate of A.

$\mathrm{adj}(A) = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

(d) Find the inverse of A.

$A^{-1} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$

Let $A = \begin{bmatrix} 3e^{2t} & 2e^{4t} \\ 4e^{2t} & -3e^{4t} \end{bmatrix}$.

(a) Find the determinant of A.

$\det(A) = \boxed{-17e^{6t}}$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} \boxed{-3e^{4t}} & \boxed{-4e^{2t}} \\ \boxed{-2e^{4t}} & \boxed{3e^{2t}} \end{bmatrix}$$

(c) Find the adjugate of A.

$$\operatorname{adj}(A) = \begin{bmatrix} \boxed{-3e^{4t}} & \boxed{-2e^{4t}} \\ \boxed{-4e^{2t}} & \boxed{3e^{2t}} \end{bmatrix}$$

(d) Find the inverse of A.

$$A^{-1} = \begin{bmatrix} \boxed{3e^{-2t}/17} & \boxed{2e^{-2t}/17} \\ \boxed{4e^{-4t}/17} & \boxed{3e^{-4t}/17} \end{bmatrix}$$

Find the area of the parallelogram with vertices at $(-3, 4)$, $(-3, 1)$, $(6, -7)$, and $(6, -10)$.

Area = [ ]

Find the area of the parallelogram with vertices at $(-3, 4)$, $(-3, 1)$, $(6, -7)$, and $(6, -10)$.

Area = 27

Find the area of the triangle with vertices $(4, -5)$, $(10, -7)$, and $(7, 3)$.

Area = ☐

Find the area of the triangle with vertices $(4, -5)$, $(10, -7)$, and $(7, 3)$.

Area = 27

Determine if v is an eigenvector of the matrix A.

?

1. $A = \begin{bmatrix} -1 & 4 & 7 \\ -1 & 4 & 7 \\ 4 & -4 & -4 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

?

2. $A = \begin{bmatrix} -1 & -1 & -2 \\ 12 & 0 & -10 \\ -6 & -1 & 3 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

?

3. $A = \begin{bmatrix} 6 & -3 & -6 \\ 0 & -3 & 0 \\ 3 & -3 & -3 \end{bmatrix}, \quad v = \begin{bmatrix} 9 \\ 7 \\ 1 \end{bmatrix}$

Determine if v is an eigenvector of the matrix A.

yes 1. $A = \begin{bmatrix} -1 & 4 & 7 \\ -1 & 4 & 7 \\ 4 & -4 & -4 \end{bmatrix}, \quad v = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

yes 2. $A = \begin{bmatrix} -1 & -1 & -2 \\ 12 & 0 & -10 \\ -6 & -1 & 3 \end{bmatrix}, \quad v = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

no 3. $A = \begin{bmatrix} 6 & -3 & -6 \\ 0 & -3 & 0 \\ 3 & -3 & -3 \end{bmatrix}, \quad v = \begin{bmatrix} 9 \\ 7 \\ 1 \end{bmatrix}$

The matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 9 & 7 & -12 \\ 8 & 6 & -11 \end{bmatrix}$$

has eigenvalues -5, 1, and 2. Find its eigenvectors.

The eigenvalue -5 has associated eigenvector $\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$.

The eigenvalue 1 has associated eigenvector $\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$.

The eigenvalue 2 has associated eigenvector $\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$.

The matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 9 & 7 & -12 \\ 8 & 6 & -11 \end{bmatrix}$$

has eigenvalues -5, 1, and 2. Find its eigenvectors.

The eigenvalue -5 has associated eigenvector $\begin{bmatrix} \boxed{0} \\ \boxed{-1} \\ \boxed{-1} \end{bmatrix}$.

The eigenvalue 1 has associated eigenvector $\begin{bmatrix} \boxed{0} \\ \boxed{-2} \\ \boxed{-1} \end{bmatrix}$.

The eigenvalue 2 has associated eigenvector $\begin{bmatrix} \boxed{1} \\ \boxed{3} \\ \boxed{2} \end{bmatrix}$.

Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -3 & 12 & -64 \\ 0 & 1 & 16 \\ 0 & 0 & 5 \end{bmatrix}.$$

The eigenvalue $\lambda_1 =$ ☐ corresponds to the eigenvector $\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$.

The eigenvalue $\lambda_2 =$ ☐ corresponds to the eigenvector $\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$.

The eigenvalue $\lambda_3 =$ ☐ corresponds to the eigenvector $\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$.

Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -3 & 12 & -64 \\ 0 & 1 & 16 \\ 0 & 0 & 5 \end{bmatrix}.$$

The eigenvalue $\lambda_1 = \boxed{-3}$ corresponds to the eigenvector $\begin{bmatrix} \boxed{-1} \\ \boxed{0} \\ \boxed{0} \end{bmatrix}$.

The eigenvalue $\lambda_2 = \boxed{1}$ corresponds to the eigenvector $\begin{bmatrix} \boxed{3} \\ \boxed{1} \\ \boxed{0} \end{bmatrix}$.

The eigenvalue $\lambda_3 = \boxed{5}$ corresponds to the eigenvector $\begin{bmatrix} \boxed{2} \\ \boxed{-4} \\ \boxed{-1} \end{bmatrix}$.

Let

$$A = \begin{bmatrix} -17 & -24 \\ 12 & 19 \end{bmatrix}.$$

Find a matrix S, a diagonal matrix D and S^{-1} such that $A = SDS^{-1}$.

$$S = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}, \quad D = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} -17 & -24 \\ 12 & 19 \end{bmatrix}.$$

Find a matrix S, a diagonal matrix D and S^{-1} such that $A = SDS^{-1}$.

$$S = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 7 & 0 \\ 0 & -5 \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}.$$

The matrix

$$C = \begin{bmatrix} 3 & 0 & 0 \\ 16 & -5 & -8 \\ -8 & 4 & 7 \end{bmatrix}$$

has two distinct eigenvalues with $\lambda_1 < \lambda_2$.

The smaller eigenvalue $\lambda_1 =$ [ ] has multiplicity [ ] and the dimension of the corresponding eigenspace is [ ].

The larger eigenvalue $\lambda_2 =$ [ ] has multiplicity [ ] and the dimension of the corresponding eigenspace is [ ].

Is the matrix C diagonalizable?

[?]

The matrix

$$C = \begin{bmatrix} 3 & 0 & 0 \\ 16 & -5 & -8 \\ -8 & 4 & 7 \end{bmatrix}$$

has two distinct eigenvalues with $\lambda_1 < \lambda_2$.

The smaller eigenvalue $\lambda_1 =$ [-1] has multiplicity [1] and the dimension of the corresponding eigenspace is [1].

The larger eigenvalue $\lambda_2 =$ [3] has multiplicity [2] and the dimension of the corresponding eigenspace is [2].

Is the matrix C diagonalizable?

[diagonalizable]

Let

$$A = \begin{bmatrix} 9 & -12 & -4 \\ 0 & -3 & 0 \\ 24 & -24 & -11 \end{bmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

$$P = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, \qquad D = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} 9 & -12 & -4 \\ 0 & -3 & 0 \\ 24 & -24 & -11 \end{bmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

$$P = \begin{bmatrix} \boxed{-1} & \boxed{-1} & \boxed{0} \\ \boxed{0} & \boxed{-1} & \boxed{-1} \\ \boxed{-2} & \boxed{0} & \boxed{3} \end{bmatrix}, \qquad D = \begin{bmatrix} \boxed{1} & \boxed{0} & \boxed{0} \\ \boxed{0} & \boxed{-3} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{-3} \end{bmatrix}.$$

Find a 2×2 matrix A such that $\begin{bmatrix} -5 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ are eigenvectors of A with eigenvalues 1 and -3, respectively.

$$A = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

Find a 2×2 matrix A such that $\begin{bmatrix} -5 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ are eigenvectors of A with eigenvalues 1 and -3, respectively.

$$A = \begin{bmatrix} \boxed{21/13} & \boxed{40/13} \\ \boxed{-12/13} & \boxed{-47/13} \end{bmatrix}$$

Show that $A = \begin{bmatrix} 1 & 2 & -4 \\ -3 & -4 & 6 \\ 0 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -2 & -2 \\ 12 & 5 & 6 \\ -6 & -3 & -4 \end{bmatrix}$ are similar matrices by finding an invertible matrix P satisfying $A = P^{-1}BP$.

$$P^{-1} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, \quad P = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}.$$

Show that $A = \begin{bmatrix} 1 & 2 & -4 \\ -3 & -4 & 6 \\ 0 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & -2 & -2 \\ 12 & 5 & 6 \\ -6 & -3 & -4 \end{bmatrix}$ are similar matrices by finding an invertible matrix P satisfying $A = P^{-1}BP$.

$$P^{-1} = \begin{bmatrix} \boxed{2} & \boxed{3} & \boxed{4} \\ \boxed{-6} & \boxed{-6} & \boxed{-7} \\ \boxed{-3} & \boxed{-2} & \boxed{-2} \end{bmatrix}, \quad P = \begin{bmatrix} \boxed{2} & \boxed{2} & \boxed{-3} \\ \boxed{-9} & \boxed{-8} & \boxed{10} \\ \boxed{6} & \boxed{5} & \boxed{-6} \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} 1 & -2 & 7 \\ -2 & 4 & -9 \\ -1 & 2 & -5 \end{bmatrix}.$$

Find the Jordan canonical form of A, where the blocks are ordered increasingly by eigenvalue and then by block size.

$$J = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} 1 & -2 & 7 \\ -2 & 4 & -9 \\ -1 & 2 & -5 \end{bmatrix}.$$

Find the Jordan canonical form of A, where the blocks are ordered increasingly by eigenvalue and then by block size.

$$J = \begin{bmatrix} \boxed{0} & \boxed{1} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{1} \\ \boxed{0} & \boxed{0} & \boxed{0} \end{bmatrix}.$$

Let

$$M = \begin{bmatrix} -3 & 1 \\ -64 & 13 \end{bmatrix}.$$

Find formulas for the entries of M^n, where n is a positive integer.

$$M^n = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}.$$

Let

$$M = \begin{bmatrix} -3 & 1 \\ -64 & 13 \end{bmatrix}.$$

Find formulas for the entries of M^n, where n is a positive integer.

$$M^n = \begin{bmatrix} \boxed{5^n - 8n5^{n-1}} & \boxed{n5^{n-1}} \\ \boxed{-64n5^{n-1}} & \boxed{8n5^{n-1} + 5^n} \end{bmatrix}.$$

Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Find a basis of $W^{\perp}$.

$$\left\{ \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \right\}.$$

Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Find a basis of $W^{\perp}$.

$$\left\{ \begin{bmatrix} \boxed{-1} \\ \boxed{-1} \\ \boxed{1} \end{bmatrix} \right\}.$$

Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Determine whether each of the following vectors is in $W^{\perp}$.

[?] 1. $v = \begin{bmatrix} -4 \\ -4 \\ 4 \end{bmatrix}$

[?] 2. $v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

[?] 3. $v = \begin{bmatrix} 7 \\ -6 \\ 3 \end{bmatrix}$

Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Determine whether each of the following vectors is in $W^{\perp}$.

yes 1. $v = \begin{bmatrix} -4 \\ -4 \\ 4 \end{bmatrix}$

yes 2. $v = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

no 3. $v = \begin{bmatrix} 7 \\ -6 \\ 3 \end{bmatrix}$

Let

$$\vec{u} = \begin{bmatrix} -1 \\ 4 \\ 1 \\ -4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -3 \\ -1 \\ -4 \end{bmatrix},$$

and let W the subspace of $\mathbb{R}^4$ spanned by $\vec{u}$ and $\vec{v}$. Find a basis of $W^\perp$, the orthogonal complement of W in $\mathbb{R}^4$.

$$\left\{ \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}, \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \right\}$$

Let

$$\vec{u} = \begin{bmatrix} -1 \\ 4 \\ 1 \\ -4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -3 \\ -1 \\ -4 \end{bmatrix},$$

and let W the subspace of $\mathbb{R}^4$ spanned by $\vec{u}$ and $\vec{v}$. Find a basis of $W^{\perp}$, the orthogonal complement of W in $\mathbb{R}^4$.

$$\left\{ \begin{bmatrix} \boxed{1} \\ \boxed{0} \\ \boxed{1} \\ \boxed{0} \end{bmatrix}, \begin{bmatrix} \boxed{28} \\ \boxed{8} \\ \boxed{0} \\ \boxed{1} \end{bmatrix} \right\}$$

Let $u_1 = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 2 \\ -13 \\ -3 \end{bmatrix}$. If $W = \mathrm{Span}\{u_1, u_2\}$, determine whether each of the following vectors is in $W^{\perp}$.

[?] 1. $v = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$

[?] 2. $v = \begin{bmatrix} -4 \\ 1 \\ -9 \end{bmatrix}$

[?] 3. $v = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

Let $u_1 = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 2 \\ -13 \\ -3 \end{bmatrix}$. If $W = \mathrm{Span}\{u_1, u_2\}$, determine whether each of the following vectors is in $W^\perp$.

no 1. $v = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$

no 2. $v = \begin{bmatrix} -4 \\ 1 \\ -9 \end{bmatrix}$

yes 3. $v = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$

Let

$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} -2 \\ -2 \\ -11 \\ -4 \end{bmatrix}.$$

Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by $\vec{x}$ and $\vec{y}$.

$$\left\{ \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}, \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \right\}.$$

Let

$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} -2 \\ -2 \\ -11 \\ -4 \end{bmatrix}.$$

Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by $\vec{x}$ and $\vec{y}$.

$$\left\{ \begin{bmatrix} \boxed{3/\sqrt{26}} \\ \boxed{1/\sqrt{26}} \\ \boxed{4/\sqrt{26}} \\ \boxed{0} \end{bmatrix}, \begin{bmatrix} \boxed{4/\sqrt{41}} \\ \boxed{0} \\ \boxed{-3/\sqrt{41}} \\ \boxed{-4/\sqrt{41}} \end{bmatrix} \right\}.$$

Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} 0 \\ -8 \\ 0 \\ 0 \end{bmatrix}$$

onto the subspace W of $\mathbb{R}^4$ spanned by

$$\begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}.$$

$$\mathrm{proj}_W(\vec{v}) = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}$$

Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} 0 \\ -8 \\ 0 \\ 0 \end{bmatrix}$$

onto the subspace W of $\mathbb{R}^4$ spanned by

$$\begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}.$$

$$\text{proj}_W(\vec{v}) = \begin{bmatrix} \boxed{-2} \\ \boxed{-6} \\ \boxed{2} \\ \boxed{2} \end{bmatrix}$$

For each of the following, factor the matrix A into a product QDQ^T where Q is orthogonal and D is diagonal.

(a) $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -4 & 2 \\ 1 & 2 & -4 \end{bmatrix}$

$$Q = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, \quad D = \begin{bmatrix} \square & 0 & 0 \\ 0 & \square & 0 \\ 0 & 0 & \square \end{bmatrix}.$$

(b) $A = \begin{bmatrix} -6 & -2 & 2 \\ -2 & -4 & 4 \\ 2 & 4 & -4 \end{bmatrix}$

$$Q = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}, \quad D = \begin{bmatrix} \square & 0 & 0 \\ 0 & \square & 0 \\ 0 & 0 & \square \end{bmatrix}.$$

For each of the following, factor the matrix A into a product QDQ^T where Q is orthogonal and D is diagonal.

(a) $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -4 & 2 \\ 1 & 2 & -4 \end{bmatrix}$

$$Q = \begin{bmatrix} \boxed{0} & \boxed{-\sqrt{3}/3} & \boxed{\sqrt{6}/3} \\ \boxed{-\sqrt{2}/2} & \boxed{\sqrt{3}/3} & \boxed{\sqrt{6}/6} \\ \boxed{\sqrt{2}/2} & \boxed{\sqrt{3}/3} & \boxed{\sqrt{6}/6} \end{bmatrix}, \quad D = \begin{bmatrix} \boxed{-6} & 0 & 0 \\ 0 & \boxed{-3} & 0 \\ 0 & 0 & \boxed{0} \end{bmatrix}.$$

(b) $A = \begin{bmatrix} -6 & -2 & 2 \\ -2 & -4 & 4 \\ 2 & 4 & -4 \end{bmatrix}$

$$Q = \begin{bmatrix} \boxed{0} & \boxed{-\sqrt{3}/3} & \boxed{\sqrt{6}/3} \\ \boxed{\sqrt{2}/2} & \boxed{\text{-}\sqrt{3}/3} & \boxed{-\sqrt{6}/6} \\ \boxed{\sqrt{2}/2} & \boxed{\sqrt{3}/3} & \boxed{\sqrt{6}/6} \end{bmatrix}, \quad D = \begin{bmatrix} \boxed{0} & 0 & 0 \\ 0 & \boxed{-10} & 0 \\ 0 & 0 & \boxed{-4} \end{bmatrix}.$$

The matrix

$$A = \begin{bmatrix} -2 & -2 & 2 \\ 0 & 2 & 0 \\ -4 & 0 & 2 \end{bmatrix}$$

has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.

The eigenvalue = [ ] has multiplicity = [ ] and the dimension of the corresponding eigenspace is [ ].

Is the matrix A defective?

[?]

The matrix

$$A = \begin{bmatrix} -2 & -2 & 2 \\ 0 & 2 & 0 \\ -4 & 0 & 2 \end{bmatrix}$$

has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.

The eigenvalue = [2] has multiplicity = [1] and the dimension of the corresponding eigenspace is [1].

Is the matrix A defective?

[defective]

The matrix

$$A = \begin{bmatrix} -2 & -1 & 0 \\ 21 & 7 & -1 \\ k & 0 & 0 \end{bmatrix}$$

has three distinct real eigenvalues if and only if

$$\boxed{} < k < \boxed{}.$$

The matrix

$$A = \begin{bmatrix} -2 & -1 & 0 \\ 21 & 7 & -1 \\ k & 0 & 0 \end{bmatrix}$$

has three distinct real eigenvalues if and only if

$$\boxed{49/27} < k < \boxed{3}.$$

Given that $\vec{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ are eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 0 \\ -4 & -2 \end{bmatrix},$$

determine the corresponding eigenvalues.

$\lambda_1 =$ ______.

$\lambda_2 =$ ______.

Given that $\vec{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ are eigenvectors of the matrix

$$A = \begin{bmatrix} 0 & 0 \\ -4 & -2 \end{bmatrix},$$

determine the corresponding eigenvalues.

$\lambda_1 =$ $\boxed{0}$.

$\lambda_2 =$ $\boxed{-2}$.

Let

$$A = \begin{bmatrix} -3 & 3 \\ -14 & 10 \end{bmatrix}.$$

Find two different diagonal matrices D and the corresponding matrix S such that $A = SDS^{-1}$.

$$D_1 = \begin{bmatrix} \square & 0 \\ 0 & \square \end{bmatrix}, \quad S_1 = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}.$$

$$D_2 = \begin{bmatrix} \square & 0 \\ 0 & \square \end{bmatrix}, \quad S_2 = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}.$$

Let

$$A = \begin{bmatrix} -3 & 3 \\ -14 & 10 \end{bmatrix}.$$

Find two different diagonal matrices D and the corresponding matrix S such that $A = SDS^{-1}$.

$$D_1 = \begin{bmatrix} \boxed{3} & 0 \\ 0 & \boxed{4} \end{bmatrix}, \quad S_1 = \begin{bmatrix} \boxed{-1} & \boxed{-3} \\ \boxed{-2} & \boxed{-7} \end{bmatrix}.$$

$$D_2 = \begin{bmatrix} \boxed{4} & 0 \\ 0 & \boxed{3} \end{bmatrix}, \quad S_2 = \begin{bmatrix} \boxed{-3} & \boxed{-1} \\ \boxed{-7} & \boxed{-2} \end{bmatrix}.$$