WeBWorK 標準問題集:線形代数学B

WeBWorK Standard Problems: Linear Algebra B

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Let
$$A = \begin{bmatrix} 5 & -2 \\ -4 & -3 \end{bmatrix}$$
.

$$det(A) =$$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} & & & & \\ & & & & \end{bmatrix}$$

(c) Find the adjugate of A.

$$\operatorname{adj}(A) = \left[\begin{array}{c|c} & & & \\ & & & \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

Let
$$A = \begin{bmatrix} 5 & -2 \\ -4 & -3 \end{bmatrix}$$
.

$$\det(A) = \boxed{-23}$$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix}$$

(c) Find the adjugate of A.

$$\operatorname{adj}(A) = \begin{bmatrix} -3 & 2 \\ 4 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3/23 & -2/23 \\ -4/23 & -5/23 \end{bmatrix}$$

$$Let A = \begin{bmatrix} -2 & -2 & -3 \\ -2 & -3 & 1 \\ -3 & 0 & 3 \end{bmatrix}.$$

$$det(A) =$$

(b) Find the matrix of cofactors of A.

$$C = \left[\begin{array}{c|c} & & & & \\ & & & & \\ & & & & \\ \end{array}\right]$$

(c) Find the adjugate of A.

$$A^{-1} = \left[\begin{array}{c|c} & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline \end{array} \right]$$

$$Let A = \begin{bmatrix} -2 & -2 & -3 \\ -2 & -3 & 1 \\ -3 & 0 & 3 \end{bmatrix}.$$

$$\det(A) = \boxed{39}$$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} -9 & 3 & -9 \\ 6 & -15 & 6 \\ -11 & 8 & 2 \end{bmatrix}$$

(c) Find the adjugate of A.

$$adj(A) = \begin{bmatrix} -9 & 6 & -11 \\ 3 & -15 & 8 \\ -9 & 6 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -3/13 & -2/13 & -11/39 \\ 1/13 & -5/13 & 8/38 \\ -3/13 & 2/13 & 2/39 \end{bmatrix}$$

Let
$$A = \begin{bmatrix} 3e^{2t} & 2e^{4t} \\ 4e^{2t} & -3e^{4t} \end{bmatrix}$$
.

$$det(A) =$$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} & & & & \\ & & & & \end{bmatrix}$$

(c) Find the adjugate of A.

$$\operatorname{adj}(A) = \left[\begin{array}{c|c} & & & \\ & & & \end{array}\right]$$

$$A^{-1} = \left[\begin{array}{c} \\ \\ \end{array} \right]$$

Let
$$A = \begin{bmatrix} 3e^{2t} & 2e^{4t} \\ 4e^{2t} & -3e^{4t} \end{bmatrix}$$
.

$$\det(A) = \boxed{-17e^{6t}}$$

(b) Find the matrix of cofactors of A.

$$C = \begin{bmatrix} -3e^{4t} \\ -2e^{4t} \end{bmatrix} \begin{bmatrix} -4e^{2t} \\ 3e^{2t} \end{bmatrix}$$

(c) Find the adjugate of A.

$$\operatorname{adj}(A) = \begin{bmatrix} -3e^{4t} & -2e^{4t} \\ -4e^{2t} & 3e^{2t} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3e^{-2t}/17 \\ 4e^{-4t}/17 \end{bmatrix} \begin{bmatrix} 2e^{-2t}/17 \\ 3e^{-4t}/17 \end{bmatrix}$$

Find the area of the parallelogram with vertices at (-3, 4), (-3, 1), (6, -7), and (6, -10).

Area =

Find the area of the parallelogram with vertices at (-3, 4), (-3, 1), (6, -7), and (6, -10).

Area =
$$\frac{27}{}$$

Find the area of the triangle with vertices (4, -5), (10, -7), and (7, 3).

Area =

Find the area of the triangle with vertices (4, -5), (10, -7), and (7, 3).

Area =
$$\frac{27}{}$$

Determine if v is an eigenvector of the matrix A.

Determine if v is an eigenvector of the matrix A.

yes 1.
$$A = \begin{bmatrix} -1 & 4 & 7 \\ -1 & 4 & 7 \\ 4 & -4 & -4 \end{bmatrix}$$
, $v = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

yes 2.
$$A = \begin{bmatrix} -1 & -1 & -2 \\ 12 & 0 & -10 \\ -6 & -1 & 3 \end{bmatrix}$$
, $v = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

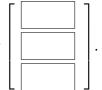
no 3.
$$A = \begin{bmatrix} 6 & -3 & -6 \\ 0 & -3 & 0 \\ 3 & -3 & -3 \end{bmatrix}$$
, $v = \begin{bmatrix} 9 \\ 7 \\ 1 \end{bmatrix}$

The matrix

$$A = \left[\begin{array}{ccc} 2 & 0 & 0 \\ 9 & 7 & -12 \\ 8 & 6 & -11 \end{array} \right]$$

has eigenvalues -5, 1, and 2. Find its eigenvectors.

The eigenvalue -5 has associated eigenvector



The eigenvalue 1 has associated eigenvector

Γ]
L	L

The eigenvalue 2 has associated eigenvector

L		

The matrix

$$A = \left[\begin{array}{rrr} 2 & 0 & 0 \\ 9 & 7 & -12 \\ 8 & 6 & -11 \end{array} \right]$$

has eigenvalues -5, 1, and 2. Find its eigenvectors.

The eigenvalue -5 has associated eigenvector

0	-	
-1		
-1		

The eigenvalue 1 has associated eigenvector

	0	
	-2	
_	-1	

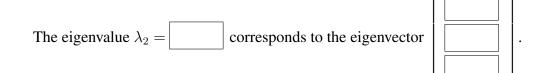
The eigenvalue 2 has associated eigenvector

1	
3	
2	

Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and corresponding eigenvectors of the matrix

$$A = \left[\begin{array}{rrr} -3 & 12 & -64 \\ 0 & 1 & 16 \\ 0 & 0 & 5 \end{array} \right].$$

The eigenvalue $\lambda_1 = \boxed{\hspace{1cm}}$ corresponds to the eigenvector		
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The eigenvalue $\lambda_3 =$ corresponds to the eigenvector		

Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and corresponding eigenvectors of the matrix

$$A = \left[\begin{array}{rrr} -3 & 12 & -64 \\ 0 & 1 & 16 \\ 0 & 0 & 5 \end{array} \right].$$

The eigenvalue $\lambda_1 = \boxed{ -3 }$ corresponds to the eigenvector $\boxed{ \begin{array}{c} -1 \\ 0 \\ \hline 0 \end{array} }$

The eigenvalue $\lambda_2 = \boxed{ \begin{array}{c} 1 \\ \hline 0 \\ \hline \end{array} }$ corresponds to the eigenvector

The eigenvalue $\lambda_3 = \boxed{\begin{array}{c} 5 \\ \hline -4 \\ \hline -1 \\ \hline \end{array}}$ corresponds to the eigenvector

$$A = \left[\begin{array}{cc} -17 & -24 \\ 12 & 19 \end{array} \right].$$

Find a matrix S, a diagonal matrix D and S^{-1} such that $A = SDS^{-1}$.

$$S = \begin{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} \end{bmatrix}, \quad D = \begin{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix} \end{bmatrix}$$

$$A = \left[\begin{array}{cc} -17 & -24 \\ 12 & 19 \end{array} \right].$$

Find a matrix S, a diagonal matrix D and S^{-1} such that $A = SDS^{-1}$.

$$S = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 7 & 0 \\ 0 & -5 \end{bmatrix}, \quad S^{-1} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

The matrix

$$C = \left[\begin{array}{rrr} 3 & 0 & 0 \\ 16 & -5 & -8 \\ -8 & 4 & 7 \end{array} \right]$$

has two distinct eigenvalues with $\lambda_1 < \lambda_2$.

The smaller eigenvalue $\lambda_1 =$	has multiplicity	and the dimension of the
corresponding eigenspace is		

The larger eigenvalue $\lambda_2 =$	=	has multiplicity	and the dimension of the
corresponding eigenspace is	.		

Is the matrix C diagonalizable?

?

The matrix

$$C = \left[\begin{array}{rrr} 3 & 0 & 0 \\ 16 & -5 & -8 \\ -8 & 4 & 7 \end{array} \right]$$

has two distinct eigenvalues with $\lambda_1 < \lambda_2$.

The smaller eigenvalue $\lambda_1 = \boxed{-1}$ has multiplicity $\boxed{1}$ and the dimension of the corresponding eigenspace is $\boxed{1}$.

The larger eigenvalue $\lambda_2= \boxed{3}$ has multiplicity $\boxed{2}$ and the dimension of the corresponding eigenspace is $\boxed{2}$.

Is the matrix C diagonalizable?

diagonalizable

$$A = \left[\begin{array}{ccc} 9 & -12 & -4 \\ 0 & -3 & 0 \\ 24 & -24 & -11 \end{array} \right].$$

Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

P =		,	D =		

$$A = \left[\begin{array}{ccc} 9 & -12 & -4 \\ 0 & -3 & 0 \\ 24 & -24 & -11 \end{array} \right].$$

Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

	-1	-1	0]		1	0
P =	0	-1	-1	,	D =	0	-3
	-2	0	3			0	0

Find a 2×2 matrix A such that $\begin{bmatrix} -5\\1 \end{bmatrix}$ and $\begin{bmatrix} 2\\-3 \end{bmatrix}$ are eigenvectors of A with eigenvalues 1 and -3, respectively.

$$A = \left[\begin{array}{c|c} & & & \\ \hline & & & \\ \end{array}\right]$$

Find a 2×2 matrix A such that $\begin{bmatrix} -5\\1 \end{bmatrix}$ and $\begin{bmatrix} 2\\-3 \end{bmatrix}$ are eigenvectors of A with eigenvalues 1 and -3, respectively.

$$A = \begin{bmatrix} 21/13 & 40/13 \\ -12/13 & -47/13 \end{bmatrix}$$

Show that
$$A = \begin{bmatrix} 1 & 2 & -4 \\ -3 & -4 & 6 \\ 0 & 0 & -1 \end{bmatrix}$$
 and $B = \begin{bmatrix} -5 & -2 & -2 \\ 12 & 5 & 6 \\ -6 & -3 & -4 \end{bmatrix}$ are similar matrices by

finding an invertible matrix P satisfying $A = P^{-1}BP$.

$P^{-1} =$			P =		
	_	_		_	

Show that
$$A = \begin{bmatrix} 1 & 2 & -4 \\ -3 & -4 & 6 \\ 0 & 0 & -1 \end{bmatrix}$$
 and $B = \begin{bmatrix} -5 & -2 & -2 \\ 12 & 5 & 6 \\ -6 & -3 & -4 \end{bmatrix}$ are similar matrices by

finding an invertible matrix P satisfying $A = P^{-1}BP$.

$$P^{-1} = \begin{bmatrix} 2 & 3 & 4 \\ -6 & -6 & -7 \\ -3 & -2 & -2 \end{bmatrix}, \quad P = \begin{bmatrix} 2 & 2 & -3 \\ -9 & -8 & 10 \\ 6 & 5 & -6 \end{bmatrix}$$

$$A = \left[\begin{array}{rrr} 1 & -2 & 7 \\ -2 & 4 & -9 \\ -1 & 2 & -5 \end{array} \right].$$

Find the Jordan canonical form of A, where the blocks are ordered increasingly by eigenvalue and then by block size.

J =		

$$A = \left[\begin{array}{rrr} 1 & -2 & 7 \\ -2 & 4 & -9 \\ -1 & 2 & -5 \end{array} \right].$$

Find the Jordan canonical form of A, where the blocks are ordered increasingly by eigenvalue and then by block size.

	0	1	0	
J =	0	0	1	
	0	0	0	

$$M = \left[\begin{array}{rr} -3 & 1 \\ -64 & 13 \end{array} \right].$$

Find formulas for the entries of ${\cal M}^n$, where n is a positive integer.

$$M^n = \left[\begin{array}{c|c} & & & \\ & & & \\ \end{array} \right].$$

$$M = \left[\begin{array}{rr} -3 & 1 \\ -64 & 13 \end{array} \right].$$

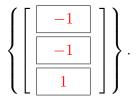
Find formulas for the entries of M^n , where n is a positive integer.

$$M^{n} = \begin{bmatrix} 5^{n} - 8n5^{n-1} \\ -64n5^{n-1} \end{bmatrix} \begin{bmatrix} n5^{n-1} \\ 8n5^{n-1} + 5^{n} \end{bmatrix}$$

Let W be the set of all vectors $\left[\begin{array}{c} x \\ y \\ x+y \end{array}\right]$ with x and y real. Find a basis of W^{\perp} .

ł			\ .

Let W be the set of all vectors $\left[\begin{array}{c} x \\ y \\ x+y \end{array}\right]$ with x and y real. Find a basis of W^{\perp} .



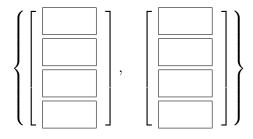
Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Determine whether each of the following vectors is in W^{\perp} .

Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Determine whether each of the following vectors is in W^{\perp} .

$$\boxed{\begin{array}{c} \mathbf{no} \\ \mathbf{3} \end{array}} 3. \ v = \left[\begin{array}{c} 7 \\ -6 \\ 3 \end{array} \right]$$

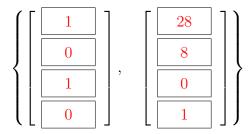
$$\vec{u} = \begin{bmatrix} -1 \\ 4 \\ 1 \\ -4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -3 \\ -1 \\ -4 \end{bmatrix},$$

and let W the subspace of \mathbb{R}^4 spanned by \vec{u} and \vec{v} . Find a basis of W^{\perp} , the orthogonal complement of W in \mathbb{R}^4 .



$$\vec{u} = \begin{bmatrix} -1 \\ 4 \\ 1 \\ -4 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -3 \\ -1 \\ -4 \end{bmatrix},$$

and let W the subspace of \mathbb{R}^4 spanned by \vec{u} and \vec{v} . Find a basis of W^{\perp} , the orthogonal complement of W in \mathbb{R}^4 .



Let
$$u_1=\begin{bmatrix} -2\\-1\\3 \end{bmatrix}$$
 and $u_2=\begin{bmatrix} 2\\-13\\-3 \end{bmatrix}$. If $W=\mathrm{Span}\{u_1,u_2\}$, determine whether each of

the following vectors is in W^{\perp} .

Let
$$u_1 = \begin{bmatrix} -2 \\ -1 \\ 3 \end{bmatrix}$$
 and $u_2 = \begin{bmatrix} 2 \\ -13 \\ -3 \end{bmatrix}$. If $W = \operatorname{Span}\{u_1, u_2\}$, determine whether each of

the following vectors is in W^{\perp} .

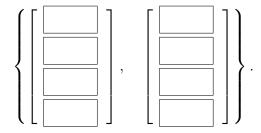
$$\begin{array}{|c|c|}\hline \mathbf{no} & 1. & v = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}$$

$$\begin{array}{|c|c|}\hline \mathbf{no} & \mathbf{2.} & v = \begin{bmatrix} -4 \\ 1 \\ -9 \end{bmatrix}$$

$$yes 3. v = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$$

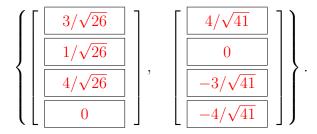
$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} -2 \\ -2 \\ -11 \\ -4 \end{bmatrix}.$$

Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of \mathbb{R}^4 spanned by \vec{x} and \vec{y} .



$$\vec{x} = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} -2 \\ -2 \\ -11 \\ -4 \end{bmatrix}.$$

Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of \mathbb{R}^4 spanned by \vec{x} and \vec{y} .



Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} 0 \\ -8 \\ 0 \\ 0 \end{bmatrix}$$

onto the subspace W of \mathbb{R}^4 spanned by

$$\begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}.$$

$$\operatorname{proj}_W(\vec{v}) = \begin{bmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \end{bmatrix}$$

Find the orthogonal projection of

$$\vec{v} = \begin{bmatrix} 0 \\ -8 \\ 0 \\ 0 \end{bmatrix}$$

onto the subspace W of \mathbb{R}^4 spanned by

$$\begin{bmatrix} -1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \end{bmatrix}.$$

$$\operatorname{proj}_{W}(\vec{v}) = \begin{bmatrix} -2 \\ -6 \\ 2 \\ 2 \end{bmatrix}$$

For each of the following, factor the matrix A into a product QDQ^T where Q is orthogonal and D is diagonal.

(a)
$$A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -4 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

(b)
$$A = \begin{bmatrix} -6 & -2 & 2 \\ -2 & -4 & 4 \\ 2 & 4 & -4 \end{bmatrix}$$

For each of the following, factor the matrix A into a product QDQ^T where Q is orthogonal and D is diagonal.

(a)
$$A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -4 & 2 \\ 1 & 2 & -4 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & -\sqrt{3}/3 & \sqrt{6}/3 \\ -\sqrt{2}/2 & \sqrt{3}/3 & \sqrt{6}/6 \\ \hline \sqrt{2}/2 & \sqrt{3}/3 & \sqrt{6}/6 \end{bmatrix}, \quad D = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

(b)
$$A = \begin{bmatrix} -6 & -2 & 2 \\ -2 & -4 & 4 \\ 2 & 4 & -4 \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & -\sqrt{3}/3 & \sqrt{6}/3 \\ \sqrt{2}/2 & -\sqrt{3}/3 & -\sqrt{6}/6 \\ \hline \sqrt{2}/2 & \sqrt{3}/3 & \sqrt{6}/6 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -4 \end{bmatrix}.$$

$$A = \left[\begin{array}{rrr} -2 & -2 & 2 \\ 0 & 2 & 0 \\ -4 & 0 & 2 \end{array} \right]$$

has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.

The eigenvalue =	has multiplicity =	and the dimension of the corre-
sponding eigenspace is		

Is the matrix A defective?

?

$$A = \left[\begin{array}{rrr} -2 & -2 & 2 \\ 0 & 2 & 0 \\ -4 & 0 & 2 \end{array} \right]$$

has one real eigenvalue. Find this eigenvalue, its multiplicity, and the dimension of the corresponding eigenspace.

The eigenvalue = $\begin{bmatrix} 2 \\ \\ \end{bmatrix}$ has multiplicity = $\begin{bmatrix} 1 \\ \\ \end{bmatrix}$ and the dimension of the corresponding eigenspace is $\begin{bmatrix} 1 \\ \\ \end{bmatrix}$.

Is the matrix A defective?

defective

$$A = \left[\begin{array}{ccc} -2 & -1 & 0 \\ 21 & 7 & -1 \\ k & 0 & 0 \end{array} \right]$$

has three distinct real eigenvalues if and only if

	< k <	
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$$A = \left[\begin{array}{ccc} -2 & -1 & 0 \\ 21 & 7 & -1 \\ k & 0 & 0 \end{array} \right]$$

has three distinct real eigenvalues if and only if

$$\boxed{49/27} < k < \boxed{3}.$$

Given that
$$\vec{v}_1=\begin{bmatrix} -1\\2 \end{bmatrix}$$
 and $\vec{v}_2=\begin{bmatrix} 0\\-1 \end{bmatrix}$ are eigenvectors of the matrix

$$A = \left[\begin{array}{cc} 0 & 0 \\ -4 & -2 \end{array} \right],$$

determine the corresponding eigenvalues.

$$\lambda_1 =$$
 .

$$\lambda_2 =$$
 .

Given that
$$\vec{v}_1=\begin{bmatrix} -1\\2 \end{bmatrix}$$
 and $\vec{v}_2=\begin{bmatrix} 0\\-1 \end{bmatrix}$ are eigenvectors of the matrix

$$A = \left[\begin{array}{cc} 0 & 0 \\ -4 & -2 \end{array} \right],$$

determine the corresponding eigenvalues.

$$\lambda_1 = \boxed{\begin{array}{c} \mathbf{0} \\ \end{array}}$$

$$\lambda_2 = \begin{vmatrix} -2 \end{vmatrix}$$

$$A = \left[\begin{array}{rr} -3 & 3 \\ -14 & 10 \end{array} \right].$$

Find two different diagonal matrices D and the corresponding matrix S such that $A = SDS^{-1}$.

$$D_1 = \begin{bmatrix} \boxed{} & 0 \\ 0 & \boxed{} \end{bmatrix}, \quad S_1 = \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} \boxed{} & 0 \\ 0 & \boxed{} \end{bmatrix}, \quad S_2 = \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}$$

$$A = \left[\begin{array}{rr} -3 & 3 \\ -14 & 10 \end{array} \right].$$

Find two different diagonal matrices D and the corresponding matrix S such that $A = SDS^{-1}$.

$$D_1 = \begin{bmatrix} \boxed{3} & 0 \\ 0 & \boxed{4} \end{bmatrix}, \quad S_1 = \begin{bmatrix} \boxed{-1} & \boxed{-3} \\ \boxed{-2} & \boxed{-7} \end{bmatrix}$$

$$D_2 = \begin{bmatrix} \boxed{4} & 0 \\ 0 & \boxed{3} \end{bmatrix}, \quad S_2 = \begin{bmatrix} \boxed{-3} & \boxed{-1} \\ \boxed{-7} & \boxed{-2} \end{bmatrix}.$$