WeBWorK 標準問題集: 解析学 A

WeBWorK Standard Problems: Calculus A

December 4, 2023

Find the limit of the sequence

$$a_n = \frac{6n^2 + 8n + 7}{2n^2 + n + 5}$$

[ ]

Find the limit of the sequence

$$a_n = \frac{6n^2 + 8n + 7}{2n^2 + n + 5}$$

3

Find the limit of the sequence

$$a_n = \frac{\cos n}{7^n}.$$

[ ]

Find the limit of the sequence

$$a_n = \frac{\cos n}{7^n}.$$

0

Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as INF. If it diverges to negative infinity, state your answer as MINF. If it diverges without being infinity or negative infinity, state your answer as DIV.

$$\lim_{n\to\infty} \frac{n^2}{e^{2n}}$$

Determine whether the sequence is divergent or convergent. If it is convergent, evaluate its limit. If it diverges to infinity, state your answer as INF. If it diverges to negative infinity, state your answer as MINF. If it diverges without being infinity or negative infinity, state your answer as DIV.

$$\lim_{n\to\infty} \frac{n^2}{e^{2n}}$$

0

Find the limit of the sequence whose terms are given by

$$a_n = n^2 \left(1 - \cos \frac{3.2}{n}\right).$$

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5.12

Find the limit of the sequence whose terms are given by

$$a_n = (e^{2n} + 6n)^{1/n}.$$

[ ]

Find the limit of the sequence whose terms are given by

$$a_n = (e^{2n} + 6n)^{1/n}.$$

e^2

Evaluate

$$\lim_{x\to\infty} \frac{\sqrt{x^4+6x^3-3}}{10x^2+9}$$

Evaluate

$$\lim_{x\to\infty} \frac{\sqrt{x^4+6x^3-3}}{10x^2+9}$$

$1/10$

Find the following limit. If the limit goes to ∞, write "inf". If the limit goes to $-\infty$, write "-inf".

$$\lim_{x\to\infty}\left(\sqrt{4x^2+x}-2x\right)$$

[ ]

Find the following limit. If the limit goes to ∞, write "inf". If the limit goes to $-\infty$, write "-inf".

$$\lim_{x\to\infty}\left(\sqrt{4x^2+x}-2x\right)$$

1/4

Evaluate the limit:

$$\lim_{x \to 4} \frac{x^2 - 16}{\sqrt{x} - 2}$$

Evaluate the limit:

$$\lim_{x\to 4}\frac{x^2-16}{\sqrt{x}-2}$$

32

Find the following limit. If the limit goes to ∞, write "inf". If the limit goes to $-\infty$, write "-inf".

$$\lim_{x\to\infty}\left[e^{-3x}\cos(3x)\right]$$

Find the following limit. If the limit goes to ∞, write "inf". If the limit goes to $-\infty$, write "-inf".

$$\lim_{x\to\infty}\left[e^{-3x}\cos(3x)\right]$$

0

Evaluate the limit

$$\lim_{x\to\frac{6}{14}} \frac{14x^2-6x}{|14x-6|}$$

Enter “INF” for ∞, “-INF” for $-\infty$, and “DNE” if the limit does not exist.

[ ]

Evaluate the limit

$$\lim_{x \to \frac{6}{14}} \frac{14x^2 - 6x}{|14x - 6|}$$

Enter “INF” for ∞, “-INF” for $-\infty$, and “DNE” if the limit does not exist.

DNE

The *signum (or sign) function*, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0. \end{cases}$$

Find each of the following limits. If the limit does not exist, enter “DNE” below.

(a) $\lim_{x\to 0^+} \operatorname{sgn} x =$ ☐

(b) $\lim_{x\to 0^-} \operatorname{sgn} x =$ ☐

(c) $\lim_{x\to 0} \operatorname{sgn} x =$ ☐

(d) $\lim_{x\to 0} |\operatorname{sgn} x| =$ ☐

The *signum (or sign) function*, denoted by sgn, is defined by

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Find each of the following limits. If the limit does not exist, enter "DNE" below.

(a) $\lim_{x\to 0^+} \operatorname{sgn} x =$ 1

(b) $\lim_{x\to 0^-} \operatorname{sgn} x =$ -1

(c) $\lim_{x\to 0} \operatorname{sgn} x =$ DNE

(d) $\lim_{x\to 0} |\operatorname{sgn} x| =$ 1

Let

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

Find $\lim_{x \to 0} f(x)$.

[ ]

Let

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ rational} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

Find $\lim_{x \to 0} f(x)$.

0

Let $f(x) = x - [x]$, where $[x]$ denotes the greatest integer that is less or equal to x. If m is an integer, find each of the following limits. If the limit does not exist, enter "DNE" below.

(a) $\lim\limits_{x \to m^-} f(x) =$ ______

(b) $\lim\limits_{x \to m^+} f(x) =$ ______

(c) $\lim\limits_{x \to m} f(x) =$ ______

Let $f(x) = x - [x]$, where $[x]$ denotes the greatest integer that is less or equal to x. If m is an integer, find each of the following limits. If the limit does not exist, enter "DNE" below.

(a) $\lim_{x \to m^-} f(x) =$ 1

(b) $\lim_{x \to m^+} f(x) =$ 0

(c) $\lim_{x \to m} f(x) =$ DNE

Evaluate

$$\lim_{\theta \to 0} \frac{\sin 3\theta \sin 7\theta}{\theta^2} .$$

Evaluate

$$\lim_{\theta \to 0} \frac{\sin 3\theta \sin 7\theta}{\theta^2}\,.$$

21

Evaluate the limit

$$\lim_{x\to 0} \frac{\tan 5x}{\sin 3x}\,.$$

Evaluate the limit

$$\lim_{x \to 0} \frac{\tan 5x}{\sin 3x}.$$

5/3

Evaluate

$$\lim_{\theta \to 0} \frac{4 \sin \theta}{\theta + 4 \tan \theta}\,.$$

[ ]

Evaluate

$$\lim_{\theta \to 0} \frac{4 \sin \theta}{\theta + 4 \tan \theta}\,.$$

$4/5$

Evaluate

$$\lim_{x\to\pi/4} \frac{5(\sin x - \cos x)}{4\cos 2x}\,.$$

[ ]

Evaluate

$$\lim_{x\to\pi/4} \frac{5(\sin x - \cos x)}{4\cos 2x}\,.$$

$-5\sqrt{2}/8$

Evaluate

$$\lim_{x\to 3} \frac{\sin(x-3)}{x^2+1x-12}.$$

Evaluate

$$\lim_{x\to 3} \frac{\sin(x-3)}{x^2+1x-12}.$$

1/7

For what value of the constant c is the function f continuous on the interval $(-\infty, \infty)$.

$$f(x) = \begin{cases} x^2 - 7, & x \leq c, \\ 8x - 23, & x > c, \end{cases}$$

$c =$ ☐

For what value of the constant c is the function f continuous on the interval $(-\infty, \infty)$.

$$f(x) = \begin{cases} x^2 - 7, & x \leq c, \\ 8x - 23, & x > c, \end{cases}$$

$c =$ 4

Find the value of the constant c that makes the following function continuous on $(-\infty, \infty)$.

$$f(s) = \begin{cases} cs + 8, & \text{if } s \in (-\infty, 5], \\ cs^2 - 8, & \text{if } s \in (5, \infty). \end{cases}$$

$c =$ ☐

Find the value of the constant c that makes the following function continuous on $(-\infty, \infty)$.

$$f(s) = \begin{cases} cs + 8, & \text{if } s \in (-\infty, 5], \\ cs^2 - 8, & \text{if } s \in (5, \infty). \end{cases}$$

$c =$ 4/5

Use the Squeeze Theorem to evaluate the following limit. If the answer is positive infinite, type “I”; if negative infinite, type “N”; and if it does not exist, type “D”.

$$\lim_{x\to\infty} \frac{\sin x}{x}$$

Use the Squeeze Theorem to evaluate the following limit. If the answer is positive infinite, type “I”; if negative infinite, type “N”; and if it does not exist, type “D”.

$$\lim_{x\to\infty} \frac{\sin x}{x}$$

0

Use the Squeeze Theorem to evaluate the limit:

$$\lim_{x\to 0} x \cos \frac{5}{x}$$

[ ]

Use the Squeeze Theorem to evaluate the limit:

$$\lim_{x\to 0} x\cos\frac{5}{x}$$

0

In this problem we consider three functions f. The first two are continuous at $x = 0$, i.e., $\lim_{x\to 0} f(x) = f(0)$. The third function is continuous from the right at $x = 0$, i.e., $\lim_{x\to 0^+} f(x) = f(0)$. In order to use the ϵ-δ definition to prove the continuity statements, one must give a definition of δ in terms of ϵ such that

$$0 < |x - 0| < \delta \quad \Longrightarrow \quad |f(x) - f(0)| < \epsilon.$$

To prove the right-continuity statement requires a definition of δ in terms of ϵ such that

$$0 < x - 0 < \delta \quad \Longrightarrow \quad |f(x) - f(0)| < \epsilon.$$

For each function in the list below, enter the number (1,2, or 3) of one of these choices

$$(1) \quad \delta = \epsilon^2; \qquad (2) \quad \delta = \epsilon; \qquad (3) \quad \delta = \sqrt{\epsilon}$$

so that your choices establish continuity of the first two functions, and right-continuity of the third function, at $x = 0$. [You may use each choice only once.]

(a) $f(x) = x$ ______

(b) $f(x) = x^2$ ______

(c) $f(x) = \sqrt{x}$ ______

In this problem we consider three functions f. The first two are continuous at $x = 0$, i.e., $\lim_{x\to 0} f(x) = f(0)$. The third function is continuous from the right at $x = 0$, i.e., $\lim_{x\to 0^+} f(x) = f(0)$. In order to use the ϵ-δ definition to prove the continuity statements, one must give a definition of δ in terms of ϵ such that

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For each function in the list below, enter the number (1,2, or 3) of one of these choices

$$(1) \quad \delta = \epsilon^2; \qquad (2) \quad \delta = \epsilon; \qquad (3) \quad \delta = \sqrt{\epsilon}$$

so that your choices establish continuity of the first two functions, and right-continuity of the third function, at $x = 0$. [You may use each choice only once.]

(a) $f(x) = x$ (2)

(b) $f(x) = x^2$ (3)

(c) $f(x) = \sqrt{x}$ (1)

Let

$$f(x) = 5^{x \tan x}.$$

Find $f'(x)$.

Let

$$f(x) = 5^{x \tan x}.$$

Find $f'(x)$.

$(\ln 5) 5^{x \tan x} (\tan x + x \sec^2 x)$

Calculate $g'(x)$, where $g(x)$ is the inverse of

$$f(x) = \frac{x}{x+2}$$

$g'(x) =$ [ ]

Calculate $g'(x)$, where $g(x)$ is the inverse of

$$f(x) = \frac{x}{x+2}$$

$g'(x) =$ $2/(x-1)^2$

If

$$f(x) = 8\sin(2x)\arcsin(x),$$

find $f'(x)$.

If

$$f(x) = 8\sin(2x)\arcsin(x),$$

find $f'(x)$.

$16\cos 2x \arcsin x + 8(\sin 2x)/\sqrt{1-x^2}$

If

$$f(x) = \left(\sin^{-1}(4x+3)\right)^3,$$

then $f'(x) =$ ______.

Note: The inverse of $\sin(x)$ can be entered as $\arcsin(x)$ or asin(x).

If

$$f(x) = \left(\sin^{-1}(4x+3)\right)^3,$$

then $f'(x) =$ $12(\sin^{-1}(4x+3))^2/\sqrt{1-(4x+3)^2}$

Note: The inverse of $\sin(x)$ can be entered as $\arcsin(x)$ or asin(x).

Let

$$f(x) = 4\sin^{-1}(x^4).$$

Find $f'(x)$.

Note: The webwork system will accept $\arcsin(x)$ or $\sin^{-1}(x)$ as the inverse of $\sin(x)$.

Let

$$f(x) = 4\sin^{-1}(x^4).$$

Find $f'(x)$.

$16x^3/\sqrt{1-x^8}$

Note: The webwork system will accept $\arcsin(x)$ or $\sin^{-1}(x)$ as the inverse of $\sin(x)$.

Let

$$f(x) = x^{7x}.$$

Use logarithmic differentiation to determine the derivative.

$f'(x) =$ [ ]

Let

$$f(x) = x^{7x}.$$

Use logarithmic differentiation to determine the derivative.

$f'(x) =$ $\boxed{x^{7x}(7 + 7\ln x)}$

If

$$f(x) = 5x^{\ln(x)},$$

then $f'(4) =$ [ ].

If

$$f(x) = 5x^{\ln(x)},$$

then $f'(4) = \boxed{5 \cdot 2^{4\ln 2}\ln 2}$.

Find a formula for $f^{(n)}(x)$ if $f(x) = e^{-5x}$.

$f^{(n)}(x) =$ [ ]

Find a formula for $f^{(n)}(x)$ if $f(x) = e^{-5x}$.

$f^{(n)}(x) =$ $\boxed{(-5)^n e^{-5x}}$

Let

$$f(x) = -12e^{-x/3}.$$

Then $f^{(9)}(1) =$ ______.

Let

$$f(x) = -12e^{-x/3}.$$

Then $f^{(9)}(1) =$ $\boxed{4/(3^8 e^{1/3})}$.

Find the 800th derivative of $f(x) = xe^{-x}$.

$f^{(800)}(x) =$ [ ]

Find the 800th derivative of $f(x) = xe^{-x}$.

$f^{(800)}(x) = \boxed{(x - 800)e^{-x}}$

Use Newton's Method with the function $f(x) = x^2 - 2$ and initial value $x_0 = 3$ to calculate x_1, x_2, x_3.

$x_1 =$ ______

$x_2 =$ ______

$x_3 =$ ______

Use Newton's Method with the function $f(x) = x^2 - 2$ and initial value $x_0 = 3$ to calculate x_1, x_2, x_3.

$x_1 =$ $11/6 \approx 1.833$

$x_2 =$ $193/132 \approx 1.462$

$x_3 =$ $72097/50952 \approx 1.415$

Integrals may have infinite limits of integration, or integrands that have singularities. Such integrals are called ”improper” even though there is nothing wrong with such integrals. They may have well defined values, in which case we say they converge, or they may not, in which case we say they diverge.

Compute the value of the following improper integral. If it is divergent, type “Diverges” or “D”.

$$\int_0^2 \frac{dx}{x^2 - 7x + 6}$$

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Compute the value of the following improper integral. If it is divergent, type “Diverges” or “D”.

$$\int_0^2 \frac{dx}{x^2 - 7x + 6}$$

D

Compute the improper integrals below. Enter the letter "D" if they diverge.

(1) $\displaystyle\int_0^1 \frac{dx}{x^2} =$ ☐

(2) $\displaystyle\int_0^1 \frac{dx}{x} =$ ☐

(3) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}} =$ ☐

(4) $\displaystyle\int_0^1 \ln x\, dx =$ ☐

(5) $\displaystyle\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}} =$ ☐

(1) $\displaystyle\int_0^1 \frac{dx}{x^2} = \boxed{}$

(2) $\displaystyle \int_0^1 \frac{dx}{x} = \boxed{}$

(3) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}} = \boxed{}$

(4) $\int_0^1 \ln x \, dx = \boxed{}$

(5) $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt{1-x^2}} = \boxed{}$

Compute the improper integrals below. Enter the letter “D” if they diverge.

(1) $\displaystyle\int_0^1 \frac{dx}{x^2} =$ D

(2) $\displaystyle\int_0^1 \frac{dx}{x} =$ D

(3) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}} =$ 2

(4) $\displaystyle\int_0^1 \ln x\, dx =$ -1

(5) $\displaystyle\int_{-1}^1 \frac{dx}{\sqrt{1-x^2}} =$ π

Give the value of the integral if it converges, and enter the letter “D” if it diverges.

(1) $\displaystyle\int_1^\infty \frac{dx}{x^2} = \boxed{}$

(2) $\displaystyle\int_1^\infty \frac{dx}{x} = \boxed{}$

(3) $\displaystyle\int_1^\infty e^{-x}\,dx = \boxed{}$

(4) $\displaystyle\int_1^\infty e^{x}\,dx = \boxed{}$

(5) $\displaystyle\int_{-\infty}^\infty \frac{dx}{1+x^2} = \boxed{}$

(6) $\displaystyle\int_{-\infty}^\pi e^{x}\,dx = \boxed{}$

(1) $\displaystyle\int_1^{\infty} \frac{dx}{x^2} = \boxed{}$

(2) $\displaystyle\int_1^{\infty} \frac{dx}{x} = \boxed{}$

(3) $\displaystyle\int_1^\infty e^{-x}\,dx = \boxed{}$

(4) $\int_1^{\infty} e^x \, dx = \boxed{}$

(5) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \boxed{}$

(6) $\displaystyle\int_{-\infty}^{\pi} e^{x}\,dx = \boxed{}$

Give the value of the integral if it converges, and enter the letter “D” if it diverges.

(1) $\displaystyle\int_1^{\infty} \frac{dx}{x^2} = \boxed{1}$

(2) $\displaystyle\int_1^{\infty} \frac{dx}{x} = \boxed{\text{D}}$

(3) $\displaystyle\int_1^{\infty} e^{-x}\,dx = \boxed{e^{-1}}$

(4) $\displaystyle\int_1^{\infty} e^{x}\,dx = \boxed{\text{D}}$

(5) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \boxed{\pi}$

(6) $\displaystyle\int_{-\infty}^{\pi} e^{x}\,dx = \boxed{e^{\pi}}$

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as “divergent”.

$$\int_6^\infty \frac{9}{(x+5)^{3/2}}\,dx$$

[ ]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as “divergent”.

$$\int_{6}^{\infty} \frac{9}{(x+5)^{3/2}}\,dx$$

$18/\sqrt{11}$

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as “divergent”.

$$\int_6^\infty \frac{\ln x}{x}\,dx$$

[ ]

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as “divergent”.

$$\int_6^{\infty} \frac{\ln x}{x}\, dx$$

D

Evaluate the following improper integral. If the integral is divergent, enter "divergent" as answer.

$$\int_{5}^{-5} \frac{dx}{|x|^{2/3}}$$

[ ]

Evaluate the following improper integral. If the integral is divergent, enter "divergent" as answer.

$$\int_{5}^{-5} \frac{dx}{|x|^{2/3}}$$

$-6 \cdot 5^{1/3}$

Evaluate the indefinite integral.

$$\int x\cos^2(7x)\,dx$$

Evaluate the indefinite integral.

$$\int x\cos^2(7x)\,dx$$

$(1/392)(98x^2 + 14x\sin 14x + \cos 14x)$

Find the exact length of the curve

$$y = \frac{x^3}{6} + \frac{1}{2x}, \qquad \frac{1}{2} \le x \le 1.$$

Arc length = $\boxed{}$

Find the exact length of the curve

$$y = \frac{x^3}{6} + \frac{1}{2x}, \qquad \frac{1}{2} \le x \le 1.$$

Arc length = $\boxed{31/48}$