WeBWorK 標準問題集: 線形代数学 A

WeBWorK Standard Problems: Linear Algebra A

December 20, 2023

Let

$$\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}.$$

Find the vectors $\vec{v} = 7\vec{x}$, $\vec{u} = \vec{x} + \vec{y}$, and $\vec{w} = 7\vec{x} + \vec{y}$.

$$\vec{v} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

Let

$$\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}.$$

Find the vectors $\vec{v} = 7\vec{x}$, $\vec{u} = \vec{x} + \vec{y}$, and $\vec{w} = 7\vec{x} + \vec{y}$.

$$\vec{v} = \begin{bmatrix} \boxed{7} \\ \boxed{21} \\ \boxed{0} \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} \boxed{3} \\ \boxed{0} \\ \boxed{1} \end{bmatrix}$$

$$\vec{w} = \begin{bmatrix} \boxed{9} \\ \boxed{18} \\ \boxed{1} \end{bmatrix}$$

The general solution to a linear system is given. Express this as a linear combination of vectors.

$$x_1 = 7 - 4s_1 - 7s_2$$
$$x_2 = -8 + 7s_1 + 1s_2$$
$$x_3 = 1 - 7s_1 + 7s_2$$
$$x_4 = -4 + 9s_1 + 4s_2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} + \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} s_1 + \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} s_2$$

The general solution to a linear system is given. Express this as a linear combination of vectors.

$$x_1 = 7 - 4s_1 - 7s_2$$
$$x_2 = -8 + 7s_1 + 1s_2$$
$$x_3 = 1 - 7s_1 + 7s_2$$
$$x_4 = -4 + 9s_1 + 4s_2$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \boxed{7} \\ \boxed{-8} \\ \boxed{1} \\ \boxed{-4} \end{bmatrix} + \begin{bmatrix} \boxed{-4} \\ \boxed{7} \\ \boxed{-7} \\ \boxed{9} \end{bmatrix} s_1 + \begin{bmatrix} \boxed{-7} \\ \boxed{1} \\ \boxed{7} \\ \boxed{4} \end{bmatrix} s_2$$

Write $\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ -4 \end{bmatrix}$.

$$\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} = \boxed{} \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix} + \boxed{} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + \boxed{} \begin{bmatrix} -4 \\ 1 \\ -4 \end{bmatrix}.$$

Write $\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$ as a linear combination of the vectors $\begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 1 \\ -4 \end{bmatrix}$.

$$\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} = \boxed{-31/30} \begin{bmatrix} 4 \\ -5 \\ 2 \end{bmatrix} + \boxed{23/15} \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + \boxed{-19/30} \begin{bmatrix} -4 \\ 1 \\ -4 \end{bmatrix}.$$

Are the vectors $\begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -16 \\ 10 \end{bmatrix}$ linearly independent? Choose

☐ linearly dependent

☐ linearly independent

If they are linearly dependent, find scalars that are not all zero such that the equation below is true. If they are linearly independent, find the only scalars that will make the equation below true.

$$\boxed{} \begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix} + \boxed{} \begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix} + \boxed{} \begin{bmatrix} -1 \\ -16 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Are the vectors $\begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ -16 \\ 10 \end{bmatrix}$ linearly independent? Choose

[✓] linearly dependent

[ ] linearly independent

If they are linearly dependent, find scalars that are not all zero such that the equation below is true. If they are linearly independent, find the only scalars that will make the equation below true.

$$\boxed{-2}\begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix} + \boxed{-3}\begin{bmatrix} -1 \\ -2 \\ 4 \end{bmatrix} + \boxed{1}\begin{bmatrix} -1 \\ -16 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Are the vectors $\begin{bmatrix} 1 \\ -5 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 4 \\ -2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -3 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -17 \\ 0 \\ 2 \\ -28 \end{bmatrix}$ linearly independent? Choose

☐ linearly dependent

☐ linearly independent

If they are linearly dependent, find scalars that are not all zero such that the equation below is true. If they are linearly independent, find the only scalars that will make the equation below true.

$$\boxed{} \begin{bmatrix} 1 \\ -5 \\ -1 \\ -1 \end{bmatrix} + \boxed{} \begin{bmatrix} -2 \\ 4 \\ -2 \\ 2 \end{bmatrix} + \boxed{} \begin{bmatrix} 4 \\ -3 \\ -1 \\ 5 \end{bmatrix} + \boxed{} \begin{bmatrix} -17 \\ 0 \\ 2 \\ -28 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Are the vectors $\begin{bmatrix} 1 \\ -5 \\ -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 4 \\ -2 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 4 \\ -3 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} -17 \\ 0 \\ 2 \\ -28 \end{bmatrix}$ linearly independent? Choose

[✓] linearly dependent

[ ] linearly independent

If they are linearly dependent, find scalars that are not all zero such that the equation below is true. If they are linearly independent, find the only scalars that will make the equation below true.

$$\boxed{-3}\begin{bmatrix} 1 \\ -5 \\ -1 \\ -1 \end{bmatrix} + \boxed{0}\begin{bmatrix} -2 \\ 4 \\ -2 \\ 2 \end{bmatrix} + \boxed{5}\begin{bmatrix} 4 \\ -3 \\ -1 \\ 5 \end{bmatrix} + \boxed{1}\begin{bmatrix} -17 \\ 0 \\ 2 \\ -28 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Which of the following sets of vectors are linearly independent? (Check the boxes for linearly independent sets.)

☐ $A = \left\{ \begin{bmatrix} 6 \\ -8 \end{bmatrix} \right\}$

☐ $B = \left\{ \begin{bmatrix} -4 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ 2 \\ 0 \end{bmatrix} \right\}$

☐ $C = \left\{ \begin{bmatrix} -9 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ -12 \\ -12 \end{bmatrix} \right\}$

☐ $D = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ -7 \end{bmatrix} \right\}$

☐ $E = \left\{ \begin{bmatrix} 8 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ -6 \\ -3 \end{bmatrix} \right\}$

☐ $F = \left\{ \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ 7 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \end{bmatrix} \right\}$

Which of the following sets of vectors are linearly independent? (Check the boxes for linearly independent sets.)

- [✓] $A = \left\{ \begin{bmatrix} 6 \\ -8 \end{bmatrix} \right\}$
- [ ] $B = \left\{ \begin{bmatrix} -4 \\ -7 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} -8 \\ 2 \\ 0 \end{bmatrix} \right\}$
- [ ] $C = \left\{ \begin{bmatrix} -9 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ -12 \\ -12 \end{bmatrix} \right\}$
- [ ] $D = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ -7 \end{bmatrix} \right\}$
- [✓] $E = \left\{ \begin{bmatrix} 8 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ -6 \\ -3 \end{bmatrix} \right\}$
- [ ] $F = \left\{ \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ 7 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \end{bmatrix} \right\}$

(1) Let W_1 be the set: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- [ ] A. W_1 is not a basis because it does not span $\mathbb{R}^3$.
- [ ] B. W_1 is not a basis because it is linearly dependent.
- [ ] C. W_1 is a basis.

(2) Let W_2 be the set: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$. Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- [ ] A. W_2 is not a basis because it is linearly dependent.
- [ ] B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- [ ] C. W_2 is a basis.

(1) Let W_1 be the set: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$. Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- [ ] A. W_1 is not a basis because it does not span $\mathbb{R}^3$.
- [ ] B. W_1 is not a basis because it is linearly dependent.
- [x] C. W_1 is a basis.

(2) Let W_2 be the set: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$. Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- [x] A. W_2 is not a basis because it is linearly dependent.
- [x] B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- [ ] C. W_2 is a basis.

Do the following sets of vectors span $\mathbb{R}^3$?

[ ] 1. $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \\ 3 \end{bmatrix}.$

[ ] 2. $\begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -7 \\ -7 \end{bmatrix}, \begin{bmatrix} 5 \\ -10 \\ -8 \end{bmatrix}.$

[ ] 3. $\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 9 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ -5 \end{bmatrix}, \begin{bmatrix} -13 \\ 39 \\ 35 \end{bmatrix}.$

[ ] 4. $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 8 \\ 4 \\ 5 \end{bmatrix}.$

Do the following sets of vectors span $\mathbb{R}^3$?

No 1. $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \\ 3 \end{bmatrix}.$

Yes 2. $\begin{bmatrix} 2 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -7 \\ -7 \end{bmatrix}, \begin{bmatrix} 5 \\ -10 \\ -8 \end{bmatrix}.$

No 3. $\begin{bmatrix} -1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 9 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ -5 \end{bmatrix}, \begin{bmatrix} -13 \\ 39 \\ 35 \end{bmatrix}.$

No 4. $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 8 \\ 4 \\ 5 \end{bmatrix}.$

Show that the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

do not span $\mathbb{R}^3$ by giving a vector not in their span.

$$\begin{bmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \end{bmatrix}$$

Show that the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$$

do not span $\mathbb{R}^3$ by giving a vector not in their span.

$$\begin{bmatrix} \boxed{1} \\ \boxed{0} \\ \boxed{2} \end{bmatrix}$$

The vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -4 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -14 \\ 0 \\ k \end{bmatrix}$$

form a basis for $\mathbb{R}^3$ if and only if $k \neq$ ______.

The vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -4 \\ 2 \\ -1 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} -14 \\ 0 \\ k \end{bmatrix}$$

form a basis for $\mathbb{R}^3$ if and only if $k \neq$ $\boxed{-4}$.

Solve the system using row operations (or elementary matrices).

$$\begin{cases} 4x + 4y + 5z &= -8 \\ 5x - 6y + 4z &= 10 \\ 5x - 5y + 6z &= 1 \end{cases}$$

$x =$ [ ]

$y =$ [ ]

$z =$ [ ]

Solve the system using row operations (or elementary matrices).

$$\begin{cases} 4x + 4y + 5z &= -8 \\ 5x - 6y + 4z &= 10 \\ 5x - 5y + 6z &= 1 \end{cases}$$

$x =$ 4

$y =$ -1

$z =$ -4

Use the Gauss-Jordan reduction to solve the following linear system:

$$\begin{cases} x_1 - x_2 - 2x_3 &= -1 \\ 4x_1 - 5x_2 - 8x_3 &= -6 \\ 3x_1 - 6x_3 &= 3 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} + s \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

Use the Gauss-Jordan reduction to solve the following linear system:

$$\begin{cases} x_1 - x_2 - 2x_3 & = -1 \\ 4x_1 - 5x_2 - 8x_3 & = -6 \\ 3x_1 - 6x_3 & = 3 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \boxed{1} \\ \boxed{2} \\ \boxed{0} \end{bmatrix} + s \begin{bmatrix} \boxed{2} \\ \boxed{0} \\ \boxed{1} \end{bmatrix}$$

Solve the system

$$\begin{cases} x_1 + x_2 \qquad\qquad = 3 \\ \qquad x_2 + x_3 \qquad = -2 \\ \qquad\qquad x_3 + x_4 = -2 \\ x_1 \qquad\qquad + x_4 = 3 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} + s \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}$$

Solve the system

$$\begin{cases} x_1+x_2 = 3 \\ x_2+x_3 = -2 \\ x_3+x_4 = -2 \\ x_1 + x_4 = 3 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \boxed{3} \\ \boxed{0} \\ \boxed{-2} \\ \boxed{0} \end{bmatrix} + s \begin{bmatrix} \boxed{-1} \\ \boxed{1} \\ \boxed{-1} \\ \boxed{1} \end{bmatrix}$$

Determine whether the following system has no solution, an infinite number of solutions or a unique solution.

[ ] 1. $\begin{cases} -7x - 3y + 3z = 3 \\ -3x - 2y - z = -4 \\ 13x + 7y - z = 0 \end{cases}$

[ ] 2. $\begin{cases} 4x + 16y + 55z = 1 \\ -x - 5y - 17z = 8 \\ 4x + 16y + 56z = 4 \end{cases}$

[ ] 3. $\begin{cases} -2x - 10y + 2z = 4 \\ -5x - 25y + 5z = 10 \\ 6x + 30y - 6z = -12 \end{cases}$

[ ] 4. $\begin{cases} -7x - 3y + 3z = 3 \\ -3x - 2y - z = -4 \\ 13x + 7y - z = 5 \end{cases}$

Determine whether the following system has no solution, an infinite number of solutions or a unique solution.

No 1. $$\begin{cases} -7x - 3y + 3z = 3 \\ -3x - 2y - z = -4 \\ 13x + 7y - z = 0 \end{cases}$$

Unique 2. $$\begin{cases} 4x + 16y + 55z = 1 \\ -x - 5y - 17z = 8 \\ 4x + 16y + 56z = 4 \end{cases}$$

Infinite 3. $$\begin{cases} -2x - 10y + 2z = 4 \\ -5x - 25y + 5z = 10 \\ 6x + 30y - 6z = -12 \end{cases}$$

Infinite 4. $$\begin{cases} -7x - 3y + 3z = 3 \\ -3x - 2y - z = -4 \\ 13x + 7y - z = 5 \end{cases}$$

Find the rank of the matrix $A = \begin{bmatrix} 9 & 1 & -3 \\ 0 & 1 & -3 \\ 36 & 4 & -15 \end{bmatrix}$.

$\text{rank}(A) =$ [ ]

Find the rank of the matrix $A = \begin{bmatrix} 9 & 1 & -3 \\ 0 & 1 & -3 \\ 36 & 4 & -15 \end{bmatrix}$.

$\text{rank}(A) =$ 3

Find the value of k for which the matrix

$$A = \begin{bmatrix} 9 & 7 & -23 \\ 2 & -4 & 6 \\ -4 & 5 & k \end{bmatrix}$$

has rank 2.

$k =$ [ ]

Find the value of k for which the matrix

$$A = \begin{bmatrix} 9 & 7 & -23 \\ 2 & -4 & 6 \\ -4 & 5 & k \end{bmatrix}$$

has rank 2.

$k =$ $\boxed{-6}$

The vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -7 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 6 \\ -2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ -8 \\ k \end{bmatrix}$$

form a basis for $\mathbb{R}^3$ if and only if $k \neq$ $\boxed{}$.

The vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ -7 \\ 0 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} -1 \\ 6 \\ -2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ -8 \\ k \end{bmatrix}$$

form a basis for $\mathbb{R}^3$ if and only if $k \neq$ $\boxed{-2}$.

Let A and B be the following matrices.

$$A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 5 \\ -7 & -7 & -5 \end{bmatrix}, \qquad B = \begin{bmatrix} -2 & 1 & 4 \\ 0 & 9 & 0 \\ 4 & -1 & -6 \end{bmatrix}$$

Perform the following operations:

$$-9A = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$A + 10B = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

$$-2A + 3B = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

Let A and B be the following matrices.

$$A = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 9 & 5 \\ -7 & -7 & -5 \end{bmatrix}, \qquad B = \begin{bmatrix} -2 & 1 & 4 \\ 0 & 9 & 0 \\ 4 & -1 & -6 \end{bmatrix}$$

Perform the following operations:

$$-9A = \begin{bmatrix} \boxed{-9} & \boxed{27} & \boxed{18} \\ \boxed{45} & \boxed{-81} & \boxed{-45} \\ \boxed{63} & \boxed{63} & \boxed{45} \end{bmatrix}$$

$$A + 10B = \begin{bmatrix} \boxed{-19} & \boxed{7} & \boxed{38} \\ \boxed{-5} & \boxed{99} & \boxed{5} \\ \boxed{33} & \boxed{-17} & \boxed{-65} \end{bmatrix}$$

$$-2A + 3B = \begin{bmatrix} \boxed{-8} & \boxed{9} & \boxed{16} \\ \boxed{10} & \boxed{9} & \boxed{-10} \\ \boxed{26} & \boxed{11} & \boxed{-8} \end{bmatrix}$$

If $A = \begin{bmatrix} 2 & -1 & -3 \\ -2 & 1 & -3 \\ -4 & 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 3 & -4 \\ 0 & 2 & -3 \\ -2 & -3 & 3 \end{bmatrix}$,

then $AB = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$

and $BA = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$.

Choose True or False: $AB = BA$ for any two square matrices A and B of the same size.

$\square$

If $A = \begin{bmatrix} 2 & -1 & -3 \\ -2 & 1 & -3 \\ -4 & 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 3 & -4 \\ 0 & 2 & -3 \\ -2 & -3 & 3 \end{bmatrix}$,

then $AB = \begin{bmatrix} 0 & 13 & -14 \\ 12 & 5 & -4 \\ 16 & 0 & 1 \end{bmatrix}$

and $BA = \begin{bmatrix} 4 & -6 & 8 \\ 8 & -7 & 0 \\ -10 & 8 & 9 \end{bmatrix}$.

Choose True or False: $AB = BA$ for any two square matrices A and B of the same size.

False

If

$$A = \begin{bmatrix} -9 \\ 4 \\ 4 \\ 9 \end{bmatrix}, \quad B = \begin{bmatrix} -8 & -1 & 8 \\ 9 & 1 & -4 \\ 3 & 1 & -6 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 6 \\ 9 & -9 & 1 \end{bmatrix},$$

then decide if each of the following operations is defined (answer yes or no)

$A + B$ [ ]

$A + C$ [ ]

$B + C$ [ ]

AB [ ]

BA [ ]

AC [ ]

CA [ ]

BC [ ]

CB [ ]

If

$$A = \begin{bmatrix} -9 \\ 4 \\ 4 \\ 9 \end{bmatrix}, \quad B = \begin{bmatrix} -8 & -1 & 8 \\ 9 & 1 & -4 \\ 3 & 1 & -6 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 6 \\ 9 & -9 & 1 \end{bmatrix},$$

then decide if each of the following operations is defined (answer yes or no)

$A + B$ no

$A + C$ no

$B + C$ no

AB no

BA no

AC no

CA no

BC no

CB yes

Let A and B be the following matrices.

$$A = \begin{bmatrix} -5 & -6 & 8 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -2 \\ -4 \\ 5 \end{bmatrix}.$$

Perform the following operation:

$$B \cdot A = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$$

Let A and B be the following matrices.

$$A = \begin{bmatrix} -5 & -6 & 8 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ -2 \\ -4 \\ 5 \end{bmatrix}.$$

Perform the following operation:

$$B \cdot A = \begin{bmatrix} 5 & 6 & -8 & -4 \\ 10 & 12 & -16 & -8 \\ 20 & 24 & -32 & -16 \\ -25 & -30 & 40 & 20 \end{bmatrix}$$

Given the matrix

$$\begin{bmatrix} 2 & 7 \\ 5 & 17 \end{bmatrix},$$

(a) does the inverse of the matrix exist? Choose Yes or No.

[ ]

(b) if your answer is yes, write the inverse here.

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

Given the matrix

$$\begin{bmatrix} 2 & 7 \\ 5 & 17 \end{bmatrix},$$

(a) does the inverse of the matrix exist? Choose Yes or No.

Yes

(b) if your answer is yes, write the inverse here.

$$\begin{bmatrix} \boxed{-17} & \boxed{7} \\ \boxed{5} & \boxed{-2} \end{bmatrix}$$

Given the matrix

$$\begin{bmatrix} 3 & 0 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix},$$

(a) does the inverse of the matrix exist? Choose Yes or No.

[ ]

(b) if your answer is yes, enter the inverse of the matrix below.

$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

Given the matrix

$$\begin{bmatrix} 3 & 0 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix},$$

(a) does the inverse of the matrix exist? Choose Yes or No.

Yes

(b) if your answer is yes, enter the inverse of the matrix below.

$$\begin{bmatrix} \boxed{1} & \boxed{1} & \boxed{-1} \\ \boxed{-1} & \boxed{-1} & \boxed{2} \\ \boxed{-2} & \boxed{-3} & \boxed{3} \end{bmatrix}$$

If

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 5 & 1 & 1 & 0 \\ -3 & -4 & -1 & 1 \end{bmatrix},$$

then $A^{-1} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$.

If

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 5 & 1 & 1 & 0 \\ -3 & -4 & -1 & 1 \end{bmatrix},$$

then $A^{-1} = \begin{bmatrix} \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} \\ \boxed{-4} & \boxed{1} & \boxed{0} & \boxed{0} \\ \boxed{-1} & \boxed{-1} & \boxed{1} & \boxed{0} \\ \boxed{-14} & \boxed{3} & \boxed{1} & \boxed{1} \end{bmatrix}.$

A square matrix is called a permutation matrix if it contains the entry 1 exactly once in each row and in each column, with all other entries being 0. All permutation matrices are invertible. Find the inverse of the permutation matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix}$$

A square matrix is called a permutation matrix if it contains the entry 1 exactly once in each row and in each column, with all other entries being 0. All permutation matrices are invertible. Find the inverse of the permutation matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} \boxed{0} & \boxed{0} & \boxed{1} & \boxed{0} \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \\ \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} \\ \boxed{0} & \boxed{1} & \boxed{0} & \boxed{0} \end{bmatrix}$$

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 4 & 12 & -49 \\ 4 & 13 & -51 \\ 1 & 3 & -12 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

(b) Use the answer from part (a) to solve the linear system

$$\begin{cases} 4x_1 + 12x_2 - 49x_3 = 4 \\ 4x_1 + 13x_2 - 51x_3 = -2 \\ x_1 + 3x_2 - 12x_3 = 2 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$$

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 4 & 12 & -49 \\ 4 & 13 & -51 \\ 1 & 3 & -12 \end{bmatrix}.$$

$$A^{-1} = \begin{bmatrix} \boxed{-3} & \boxed{-3} & \boxed{25} \\ \boxed{-3} & \boxed{1} & \boxed{8} \\ \boxed{-1} & \boxed{0} & \boxed{4} \end{bmatrix}$$

(b) Use the answer from part (a) to solve the linear system

$$\begin{cases} 4x_1 + 12x_2 - 49x_3 = 4 \\ 4x_1 + 13x_2 - 51x_3 = -2 \\ x_1 + 3x_2 - 12x_3 = 2 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \boxed{44} \\ \boxed{2} \\ \boxed{4} \end{bmatrix}$$

Find the determinant of the matrix

$$A = \begin{bmatrix} -1 & 3 \\ 9 & 2 \end{bmatrix}.$$

$\det(A) =$ [ ]

Find the determinant of the matrix

$$A = \begin{bmatrix} -1 & 3 \\ 9 & 2 \end{bmatrix}.$$

$\det(A) =$ -29

Find the determinant of the matrix

$$\begin{bmatrix} 3 & 5 & -3 \\ -1 & 0 & 1 \\ 0 & 3 & -2 \end{bmatrix}.$$

determinant $=$ $\boxed{}$

Find the determinant of the matrix

$$\begin{bmatrix} 3 & 5 & -3 \\ -1 & 0 & 1 \\ 0 & 3 & -2 \end{bmatrix}.$$

determinant $=$ $\boxed{-10}$

Given the matrix

$$A = \begin{bmatrix} 4 & 1 & 4 \\ 3 & 1 & -2 \\ -3 & 1 & 5 \end{bmatrix},$$

find its determinant.

The determinant is $\boxed{}$.

Given the matrix

$$A = \begin{bmatrix} 4 & 1 & 4 \\ 3 & 1 & -2 \\ -3 & 1 & 5 \end{bmatrix},$$

find its determinant.

The determinant is $\boxed{\color{red}{43}}$.

If

$$A = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 5 & -4 & 0 & 0 & 0 \\ -9 & -9 & 9 & 0 & 0 \\ 5 & -6 & 7 & -1 & 0 \\ 8 & 6 & -8 & -1 & 4 \end{bmatrix},$$

then $\det(A) = \boxed{}$.

If

$$A = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 5 & -4 & 0 & 0 & 0 \\ -9 & -9 & 9 & 0 & 0 \\ 5 & -6 & 7 & -1 & 0 \\ 8 & 6 & -8 & -1 & 4 \end{bmatrix},$$

then $\det(A) =$ $\boxed{864}$.

If

$$A = \begin{bmatrix} -3-2i & -1-3i \\ -2+3i & -2+3i \end{bmatrix},$$

then $|A| =$ ☐.

If

$$A = \begin{bmatrix} -3-2i & -1-3i \\ -2+3i & -2+3i \end{bmatrix},$$

then $|A| = \boxed{1-8i}$.

A square matrix is called a permutation matrix if each row and each column contains exactly one entry 1, with all other entries being 0. An example is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find the determinant of this matrix.

$\det(P) =$ [ ]

A square matrix is called a permutation matrix if each row and each column contains exactly one entry 1, with all other entries being 0. An example is

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

Find the determinant of this matrix.

$\det(P) =$ 1

Find the determinant of the matrix

$$B = \begin{bmatrix} 0 & -2 & 3 \\ -3 & 3 & 5 \\ 0 & -2 & 5 \end{bmatrix}.$$

$\det(B) =$ ______

Find the determinant of the matrix

$$B = \begin{bmatrix} 0 & -2 & 3 \\ -3 & 3 & 5 \\ 0 & -2 & 5 \end{bmatrix}.$$

$\det(B) = \boxed{-12}$

Find the determinant of the matrix

$$M = \begin{bmatrix} 5 & -6 & 8 \\ 0 & -5 & -7 \\ 0 & 0 & 1 \end{bmatrix}.$$

$\det(M) =$ ____

Find the determinant of the matrix

$$M = \begin{bmatrix} 5 & -6 & 8 \\ 0 & -5 & -7 \\ 0 & 0 & 1 \end{bmatrix}.$$

$\det(M) =$ $\boxed{-25}$

If

$$\det\begin{bmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{bmatrix} = -3, \quad \text{and} \quad \det\begin{bmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{bmatrix} = 5,$$

then

$$\det\begin{bmatrix} a & 8 & d \\ b & 8 & e \\ c & 8 & f \end{bmatrix} = \boxed{}$$

$$\det\begin{bmatrix} a & 5 & d \\ b & 8 & e \\ c & 11 & f \end{bmatrix} = \boxed{}.$$

If

$$\det\begin{bmatrix} a & 1 & d \\ b & 1 & e \\ c & 1 & f \end{bmatrix} = -3, \quad \text{and} \quad \det\begin{bmatrix} a & 1 & d \\ b & 2 & e \\ c & 3 & f \end{bmatrix} = 5,$$

then

$$\det\begin{bmatrix} a & 8 & d \\ b & 8 & e \\ c & 8 & f \end{bmatrix} = \boxed{-24}$$

$$\det\begin{bmatrix} a & 5 & d \\ b & 8 & e \\ c & 11 & f \end{bmatrix} = \boxed{9}.$$

The determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

is ______.

The determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

is $\boxed{0}$.

The determinant of the matrix

$$A = \begin{bmatrix} 2 & -9 & 0 & -6 \\ 5 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ -1 & 4 & -9 & -6 \end{bmatrix}$$

is ______.

The determinant of the matrix

$$A = \begin{bmatrix} 2 & -9 & 0 & -6 \\ 5 & 1 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ -1 & 4 & -9 & -6 \end{bmatrix}$$

is 810 .

Evaluate the following 4×4 determinant. Use the properties of determinants to your advantage.

$$\begin{vmatrix} 3 & 12 & 1 & 1 \\ 1 & 1 & 6 & -1 \\ 0 & 0 & 1 & 0 \\ 10 & 2 & 5 & -9 \end{vmatrix}$$

Answer: [ ]

Evaluate the following 4×4 determinant. Use the properties of determinants to your advantage.

$$\begin{vmatrix} 3 & 12 & 1 & 1 \\ 1 & 1 & 6 & -1 \\ 0 & 0 & 1 & 0 \\ 10 & 2 & 5 & -9 \end{vmatrix}$$

Answer: -41

Given

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -3,$$

find the following determinants.

$$\det \begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix} = \boxed{}$$

$$\det \begin{bmatrix} a & b & c \\ -8d+a & -8e+b & -8f+c \\ g & h & i \end{bmatrix} = \boxed{}$$

$$\det \begin{bmatrix} -8d+a & -8e+b & -8f+c \\ d & e & f \\ g & h & i \end{bmatrix} = \boxed{}$$

Given

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -3,$$

find the following determinants.

$$\det \begin{bmatrix} g & h & i \\ a & b & c \\ d & e & f \end{bmatrix} = \boxed{-3}$$

$$\det \begin{bmatrix} a & b & c \\ -8d+a & -8e+b & -8f+c \\ g & h & i \end{bmatrix} = \boxed{24}$$

$$\det \begin{bmatrix} -8d+a & -8e+b & -8f+c \\ d & e & f \\ g & h & i \end{bmatrix} = \boxed{-3}$$

Suppose that a 4×4 matrix A with rows $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, and $\vec{v}_4$ has determinant $\det A = 9$. Find the following determinants.

$$\det \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ 7\vec{v}_4 \end{bmatrix} = \boxed{}$$

$$\det \begin{bmatrix} \vec{v}_2 \\ \vec{v}_1 \\ \vec{v}_4 \\ \vec{v}_3 \end{bmatrix} = \boxed{}$$

$$\det \begin{bmatrix} \vec{v}_1 + 4\vec{v}_3 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{bmatrix} = \boxed{}$$

Suppose that a 4×4 matrix A with rows $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, and $\vec{v}_4$ has determinant $\det A = 9$. Find the following determinants.

$$\det \begin{bmatrix} \vec{v}_1 \\ \vec{v}_2 \\ \vec{v}_3 \\ 7\vec{v}_4 \end{bmatrix} = \boxed{63}$$

$$\det \begin{bmatrix} \vec{v}_2 \\ \vec{v}_1 \\ \vec{v}_4 \\ \vec{v}_3 \end{bmatrix} = \boxed{9}$$

$$\det \begin{bmatrix} \vec{v}_1 + 4\vec{v}_3 \\ \vec{v}_2 \\ \vec{v}_3 \\ \vec{v}_4 \end{bmatrix} = \boxed{9}$$

Find the determinant of the matrix

$$M = \begin{bmatrix} -2 & 0 & 0 & 1 & 0 \\ -1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & -2 & -2 \\ 0 & -1 & -3 & 0 & 0 \end{bmatrix}.$$

$\det(M) =$ ______

Find the determinant of the matrix

$$M = \begin{bmatrix} -2 & 0 & 0 & 1 & 0 \\ -1 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 & 3 \\ 0 & 0 & 0 & -2 & -2 \\ 0 & -1 & -3 & 0 & 0 \end{bmatrix}.$$

$\det(M) = \boxed{-36}$

Find the determinant of the matrix

$$M = \begin{bmatrix} 3 & 0 & 0 & -3 \\ -3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -2 & -2 & 0 \end{bmatrix}.$$

$\det(M) =$ ______

Find the determinant of the matrix

$$M = \begin{bmatrix} 3 & 0 & 0 & -3 \\ -3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -2 & -2 & 0 \end{bmatrix}.$$

$\det(M) =$ 24

The determinant of the matrix

$$A = \begin{bmatrix} 0 & -2 & -1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 5 & -9 & -9 & -5 & -3 \\ 0 & -7 & 2 & 0 & 9 \end{bmatrix}$$

is ______.

Hint: Find a good row or column and expand by minors.

The determinant of the matrix

$$A = \begin{bmatrix} 0 & -2 & -1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 5 & -9 & -9 & -5 & -3 \\ 0 & -7 & 2 & 0 & 9 \end{bmatrix}$$

is | 270 |.

Hint: Find a good row or column and expand by minors.

The determinant of the matrix

$$A = \begin{bmatrix} 9 & -7 & 0 & 8 \\ 2 & -9 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 7 & 2 & -8 & -7 \end{bmatrix}$$

is $\boxed{}$.

Hint: Find a good row or column and expand by minors.

The determinant of the matrix

$$A = \begin{bmatrix} 9 & -7 & 0 & 8 \\ 2 & -9 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 7 & 2 & -8 & -7 \end{bmatrix}$$

is $\boxed{-256}$.

Hint: Find a good row or column and expand by minors.

Let

$$A = \begin{bmatrix} -5 & -7 & -4 \\ -6 & 1 & 3 \end{bmatrix}.$$

Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(\vec{x}) = A\vec{x}$. Find the images of

$$\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

under T.

$$T(\vec{u}) = \begin{bmatrix} \square \\ \square \end{bmatrix}$$

$$T(\vec{v}) = \begin{bmatrix} \square \\ \square \end{bmatrix}$$

Let

$$A = \begin{bmatrix} -5 & -7 & -4 \\ -6 & 1 & 3 \end{bmatrix}.$$

Define the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(\vec{x}) = A\vec{x}$. Find the images of

$$\vec{u} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

under T.

$$T(\vec{u}) = \begin{bmatrix} \boxed{7} \\ \boxed{-12} \end{bmatrix}$$

$$T(\vec{v}) = \begin{bmatrix} \boxed{-5a - 7b - 4c} \\ \boxed{-6a + b + 3c} \end{bmatrix}$$

If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix},$$

then

$$T\left(\begin{bmatrix} -5 \\ -4 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} \boxed{} \\ \boxed{} \\ \boxed{} \end{bmatrix}$$

If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \quad T\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix},$$

then

$$T\left(\begin{bmatrix} -5 \\ -4 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} \boxed{-18} \\ \boxed{11} \\ \boxed{3} \end{bmatrix}$$

Find the matrix M of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -8x_1 + x_2 + (-9)\, x_3 \\ -5x_1 + 9x_3 \end{bmatrix}.$$

$$M = \begin{bmatrix} \square & \square & \square \\ \square & \square & \square \end{bmatrix}$$

Find the matrix M of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -8x_1 + x_2 + (-9)\,x_3 \\ -5x_1 + 9x_3 \end{bmatrix}.$$

$$M = \begin{bmatrix} \boxed{-8} & \boxed{1} & \boxed{-9} \\ \boxed{-5} & \boxed{0} & \boxed{9} \end{bmatrix}$$

Let

$$A = \begin{bmatrix} -2 & -3 \\ -4 & -6 \end{bmatrix}.$$

Find bases for the kernel and image of $T(\vec{x}) = A\vec{x}$.

A basis for the kernel of A is $\left\{ \begin{bmatrix} \square \\ \square \end{bmatrix} \right\}$.

A basis for the image of A is $\left\{ \begin{bmatrix} \square \\ \square \end{bmatrix} \right\}$.

Let

$$A = \begin{bmatrix} -2 & -3 \\ -4 & -6 \end{bmatrix}.$$

Find bases for the kernel and image of $T(\vec{x}) = A\vec{x}$.

A basis for the kernel of A is $\left\{ \begin{bmatrix} \boxed{-3} \\ \boxed{2} \end{bmatrix} \right\}$.

A basis for the image of A is $\left\{ \begin{bmatrix} \boxed{-1} \\ \boxed{-2} \end{bmatrix} \right\}$.

Let

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix}.$$

Find a basis for the image of A (or, equivalently, for the linear transformation $T(x) = Ax$).

A basis for the image of A is $\left\{ \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix}, \begin{bmatrix} \square \\ \square \\ \square \\ \square \end{bmatrix} \right\}$.

Let

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix}.$$

Find a basis for the image of A (or, equivalently, for the linear transformation $T(x) = Ax$).

A basis for the image of A is $\left\{ \begin{bmatrix} \boxed{1} \\ \boxed{1} \\ \boxed{-1} \\ \boxed{0} \end{bmatrix}, \begin{bmatrix} \boxed{-4} \\ \boxed{-1} \\ \boxed{1} \\ \boxed{-3} \end{bmatrix} \right\}.$

Let

$$A = \begin{bmatrix} -3 & -2 & 4 \\ -9 & -6 & 12 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for the kernel of A (or, equivalently, for the linear transformation $T(x) = Ax$).

A basis for the kernel of A is $\left\{ \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}, \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix} \right\}$.

Let

$$A = \begin{bmatrix} -3 & -2 & 4 \\ -9 & -6 & 12 \\ 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for the kernel of A (or, equivalently, for the linear transformation $T(x) = Ax$).

A basis for the kernel of A is $\left\{ \begin{bmatrix} \boxed{4} \\ \boxed{0} \\ \boxed{3} \end{bmatrix}, \begin{bmatrix} \boxed{-2} \\ \boxed{3} \\ \boxed{0} \end{bmatrix} \right\}$.

Let

$$A = \begin{bmatrix} 4 & 8 \\ 6 & 15 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} -20 \\ -36 \\ -3 \end{bmatrix}.$$

A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T(x) = Ax$. Find an $\vec{x}$ in $\mathbb{R}^2$ whose image under T is $\vec{b}$.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix}$$

Let

$$A = \begin{bmatrix} 4 & 8 \\ 6 & 15 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} -20 \\ -36 \\ -3 \end{bmatrix}.$$

A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by $T(x) = Ax$. Find an $\vec{x}$ in $\mathbb{R}^2$ whose image under T is $\vec{b}$.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \boxed{-1} \\ \boxed{-2} \end{bmatrix}$$

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T(\vec{x}) = \begin{bmatrix} 3 & 5 \\ 2 & -1 \end{bmatrix} \vec{x}.$$

Find the matrix M of the inverse linear transformation T^{-1}.

$$M = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$T(\vec{x}) = \begin{bmatrix} 3 & 5 \\ 2 & -1 \end{bmatrix} \vec{x}.$$

Find the matrix M of the inverse linear transformation T^{-1}.

$$M = \begin{bmatrix} \boxed{1/13} & \boxed{5/13} \\ \boxed{2/13} & \boxed{-3/13} \end{bmatrix}$$